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Generation of a Crowned Pinion Tooth Surface by a Surface of Revolution

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SUMMARY

A method of generating crowned pinion tooth surfaces using a surface of revolution is developed. The crowned pinion meshes with a regular involute gear and has a prescribed parabolic type of transmission errors when the gears operate in the aligned mode. When the gears are misaligned the transmission error remains parabolic with the maximum level still remaining very small (less than 0.34 arc sec for the numerical examples). Tooth contact analysis (TCA) is used to simulate the conditions of meshing, determine the transmission error, and determine the bearing contact.

INTRODUCTION

Parallel axis gearing is very sensitive to misalignment of the gear rotation axes. Misalignment can cause edge loading of the teeth and thus premature failure of the gear components. Crowning one or both of the gear pairs can be used. The surface can be modified in the longitudinal direction or in the profile direction, or a combination of longitudinal and profile modification can be used to modify the surface. All modifications are directed at finding a satisfactory amount of crowning for a given degree of misalignment.

Another condition, kinematic error (transmission error), results when gears are crowned and their axes operate in a misaligned mode. This condition can increase the gear mesh generated noise. The kinematic error that can result can be severe enough to cause an interruption or discontinuity during the meshing process. The discontinuity causes impacting of the teeth instead of the relatively smooth action that is typically found in parallel axis gearing operating under light load conditions.

A method for crowned gear generation is presented. The method only requires that the pinion be crowned. The pinion tooth surface is modified in the profile and longitudinal directions. The gear that is in mesh with the pinion is a regular involute.

The pinion crowning method used is as follows:

(1) The pinion generating tool is provided with a surface of revolution that is slightly modified from a cone surface.

(2) The pinion tooth surface and the tool generating surface are in line contact at every instant.

(3) The relationship between the motions of the tool and the pinion in the process of generation is the same as that between the rack cutter and the generated spur gear.

(4) The surface of revolution of the generating tool is rotated about its axis only to provide the desired velocity of cutting.

The synthesis of spur gears generated by this method is based on determining the appropriate principal curvatures of the generating surface.

GENERATION METHOD

The generation method to be discussed has the following requirements: (1) obtain a localized bearing contact and (2) reduce the gear sensitivity to misalignment. The generation method shown in figure 1 (coned surface for generating pinion, rack cutter for generating gear) satisfies only the first requirement - localization of the bearing contact. However, it is preferable to start discussions with this generation method and then to extend the analysis to a more sophisticated generation method to satisfy both requirements.

The basic idea of the first generation method is the application of two rigidly connected generating surfaces - a plane and a cone. The plane is the surface of a regular rack cutter. Both generating surfaces are in tangency along a straight line - the generatrix of the cone surface. The cone generates the pinion tooth surface, and the plane generates the gear tooth surface. In the process of generation the rack cutter and the cone perform a translational motion while the pinion and the gear rotate about their axes (fig. 2). The rotation of the cone about its axis c-c is not related to other motions that must be provided to generate the tooth surface. The angular velocity of the cone depends only on the desired velocity of cutting. The tool for crowning the pinion can be designed as a grinding wheel or as a shaver. The pinion is generated one side at a time.

The described process of pinion crowning by a cone provides an involute shape of the pinion tooth surface in its middle section. The crowned pinion and the involute gear, if they are not misaligned, can transform rotation without transmission errors and their bearing contact can be localized. However, these gears when misaligned will transform rotation with transmission errors of type 1 (fig. 3(a)). To avoid the discontinuity during meshing, a surface of revolution must be used instead of a cone surface. This surface is slightly modified from a regular cone surface, and its application for crowning provides kinematical errors of type 2 (fig. 3(b)). The generated pinion tooth shape in the middle cross section is no longer an involute curve. The pinion and the gear, even if they are not misaligned, transform rotation with a low level of transmission errors that resemble a parabolic function. If the gears are misaligned the transmission errors are still a parabolic function.

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COORDINATE SYSTEMS AND TOOL SURFACES

While considering the generation of gear tooth surfaces, we use coordinate systems S_1, S_2, S_c, and S_f (fig. 2) where S_1 and S_2 are rigidly connected to the pinion and gear being generated; S_c is rigidly connected to the cutters that perform their translational motion as one rigid body; and S_f is the fixed coordinate system. The tool displacement s and the angles of gear rotation are related as follows:

$$s = r_1\phi_P = r_2\phi_G$$

where r_1 and r_2 are the radii of the pinion and gear pitch circles, respectively.

While considering the meshing of the generated gears, we will use the same coordinate systems S_1, S_2, and S_f (fig. 4). In order to put the generated surfaces (fig. 2) in contact, the direction of rotation of the pinion and gear are reversed (fig. 4). Angles ϕ_P and ϕ_G are the angles of pinion and gear rotation while they are generated by the tools (fig. 2); ϕ_1 and ϕ_2 are the angles of pinion and gear rotation while the gears are in direct meshing with each other. Angles ϕ_G and ϕ_P are related by a linear function, but the function $\phi_2(\phi_1)$ is a nonlinear one. The deviation of function $\phi_2(\phi_1)$ from the linear one depends on the deviation of the tool surface of revolution from a regular cone surface.

Figure 5 shows three different cases of generating tool surfaces. In case 1 (fig. 5(a)), both rigidly connected generating surfaces Σ_P and Σ_G are planar (rack cutter) and coincident with each other. The generated gears will be regular involute surfaces. Thus, the pinion is not crowned, and the bearing contact is not localized.

In case 2 (fig. 5(b)), Σ_P is a regular cone surface and Σ_G is a plane. Surfaces Σ_P and Σ_G are in contact at the generatrix of the cone. The tool surface Σ_P generates the crowned pinion tooth surface, and Σ_G generates the regular involute gear tooth surface. The bearing contact of the pinion and gear tooth surfaces is localized, but a predesigned deviation of function $\phi_2(\phi_1)$ from the linear function

$$\phi_2^0 = \frac{N_1}{N_2}\,\phi_1 \tag{1}$$

cannot be obtained where N_1 and N_2 are the tooth numbers of the pinion and gear, respectively.

In case 3 (fig. 5(c)), the pinion generating surface Σ_P is a surface of revolution, and the gear generating surface Σ_G is again a plane. The tool surfaces are in tangency at a point instead of at a line as in case 2. The pinion is provided with a crowned tooth surface. The bearing contact of the gear tooth surfaces is localized, and function $\phi_2(\phi_1)$ can deviate from the linear function (eq. (1)) by the appropriate value of the radius of the arc of the surface of revolution. This arc is the axial section of the surface of revolution.

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LOCAL SYNTHESIS

The gear local synthesis must provide the following conditions of meshing at the mean contact point P (fig. 4) and its neighborhood:

(1) The gear tooth surfaces are in contact at P.

(2) The instantaneous gear ratio is not constant but is represented by a function $m_{21}(\phi_1)$ where

$$m_{21}(\phi_1) = \frac{d}{d\phi_1} [\phi_2(\phi_1)] \tag{2}$$

(3) The instantaneous value of $m_{21}(\phi_1)$ at $\phi_1 = 0$ must be equal to the prescribed value

$$m_{21}(0) = \frac{N_1}{N_2} \tag{3}$$

(4) The function $\phi_2(\phi_1)$ is represented as

$$\phi_2(\phi_1) = \frac{N_1}{N_2} \phi_1 + \Delta\phi_2(\phi_1) \tag{4}$$

where $\Delta\phi_2(\phi_1)$ is the predesigned function. Assuming that $\Delta\phi_2(\phi_1)$ is a parabolic function given as

$$\Delta\phi_2(\phi_1) = -a\phi_1^2 \tag{5}$$

we obtain, when $\phi_1 = 0$,

$$m_{21}(0) = \frac{d}{d\phi_1} [\phi_2(\phi_1)] = \frac{N_1}{N_2} \tag{6}$$

and

$$\frac{dm_{21}(0)}{d\phi_1} = m_{21}'(0) = \frac{d^2}{d\phi_1^2} [\phi_2(\phi_1)] = -2a \tag{7}$$

This means that a predesigned parabolic function for transmission errors satisfies the requirement that the instantaneous gear ratio $m_{21}(0)$ is equal to the given value (eq. (6)) and provides a negative value for the derivative $m_{21}'(0)$.

(5) When the generating surface of revolution is used, the existence of the parabolic function for transmission errors can only be guaranteed in the neighborhood of the mean contact point. The magnitude of $m_{21}(0)$ can be controlled by choosing the appropriate curvature radius ρ of the axial section of the surface of revolution. The determination of ρ, which is based on the equation given in references 1 and 2, relates the principal curvatures and directions and $m_{21}'(0)$. For the case of crowned spur gears, this equation is represented as

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$$m'_{21} = -\left(1 + \frac{N_1}{N_2}\right) \tan \psi_c \frac{\dfrac{1}{\rho}}{\left(1 + \dfrac{N_1}{N_2}\right) \dfrac{1}{r_1 \sin \psi_c} + \dfrac{1}{\rho}} \tag{8}$$

where ψ_c is the pressure angle and ρ the radius of the axial section arc of generating surface (fig. 7).

CROWNED PINION TOOTH SURFACE EQUATIONS

Consider that the tool surface (a surface of revolution) is represented in an auxiliary coordinate system by the vector function $r_a(\Theta_p, \beta)$ where Θ_p and β are the surface coordinates. Matrix $[M_{ca}]$ describes the installment of the tool system S_a in coordinate system S_c. Using the coordinate transformation S_a to S_c given by

$$[r_c] = [M_{ca}] [r_a] \tag{9}$$

we can represent the tool surface in S_c by the vector function $r_a(\Theta_p, \beta)$.

While the coordinate system S_c translates, the pinion being generated rotates (fig. 2). The installment of the tool with respect to the pinion is shown in figure 6. Equations of the generated pinion are represented as

$$[r_1] = [M_{1c}] [r_c] \tag{10}$$

$$f(\phi_p, \Theta_p, \beta) = N^{(c)} \cdot V^{(cP)} = 0 \tag{11}$$

Equation (10) describes coordinate transformation in transition from S_c to S_1. Equation (11) is the equation of meshing, $N^{(c)}$ is the tool surface normal, and $V^{(cP)}$ is the relative velocity of the tool with respect to the pinion.

Equations (10) and (11) represent the pinion tooth surface in the three-parameter form with the additional relation between the parameters. Thus,

$$\left. \begin{array}{l} r_1 = r_1(\Theta_p, \beta, \phi_p) \\[2ex] f(\phi_p, \Theta_p, \beta) = 0 \end{array} \right\} \tag{12}$$

By eliminating one of these parameters, we are able to represent the pinion tooth surface in the traditional two-parameter form. More details of pinion surface generation are contained in the appendix.

TCA PROGRAM

TCA programs have been developed based on the principles presented in reference 3. These programs simulate the conditions of meshing and determine the transmission errors caused by gear misalignment.

A numerical example is given for two types of misalignment. The following conditions are given: pinion and gear tooth numbers, $N_1 = 20$, $N_2 = 40$; diametral pitch, $P_n = 10$ in.$^{-1}$; pressure angle, $\psi_c = 20°$; half apex angle of auxiliary cone, $\alpha = 80°$; height of auxiliary cone, $d = 0.176$ in.; radius of generatrix arc, $\rho = 500$ in. (figs. 7 and 8). The developed TCA program has been applied to evaluate the kinematic error for the following:

(1) The change of the center distance is $\Delta C/C = 1$ percent, where $C = (N_1 + N_2)/2P_n$. The gear axes are not parallel but they cross, and the twist angle is 5 to 10 arc min. The kinematic error is of type 2 (fig. 3(b)). The maximum values of kinematic error for these cases are 0.30 and 0.31 arc sec, respectively.

(2) The change of center distance is $\Delta C/C = 1$ percent, where $C = (N_1 + N_2)/2P_n$. The gear axes are not parallel but they intersect, and the intersecting angle is 5 and 10 arc min. The kinematic error is also type 2 (fig. 3(b)). Corresponding maximum values of the kinematic error are 0.34 and 0.32 arc sec, respectively.

The results of this numerical example illustrate that the proposed method for generating a crowned pinion surface can avoid the appearance of a lead function of transmission errors and can reduce the level of these errors.

CONCLUSIONS

A method for generating a crowned pinion tooth surface using a surface of revolution has been developed. The meshing gear is generated by using a rack cutter. The following are the conclusions of this study:

1. A surface of revolution is used to generate a crowned pinion surface. The pinion meshing with a regular involute gear provides a prescribed parabolic function of transmission errors when there is no misalignment.

2. When the modified gears are misaligned, the transmission errors remain a parabolic function without discontinuities.

3. The maximum level of kinematic error, due to misalignment, remains low.

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APPENDIX - EQUATIONS OF PINION GENERATING SURFACE AND PINION TOOTH SURFACE

As shown in figure 7(a), arc B and straight line C are the generatrices of the revolute surface and the cone. Arc B and its normal can be represented in coordinate system S_e as follows:

$$[r_e] = \begin{bmatrix} \rho[\cos(\alpha + \beta) - \cos\alpha] + a_1 \sin\alpha \\ \rho[\sin(\alpha + \beta) - \sin\alpha] + a_2 \cos\alpha \\ 0 \\ 1 \end{bmatrix} \tag{13a}$$

$$[r_e] = \begin{bmatrix} a_1 \sin\alpha - 2\rho \sin(\beta/2) \sin(\alpha + \beta/2) \\ a_2 \cos\alpha + 2\rho \sin(\beta/2) \cos(\alpha + \beta/2) \\ 0 \\ 1 \end{bmatrix} \tag{13b}$$

$$[n_e] = \begin{bmatrix} \cos(\alpha + \beta) \\ \sin(\alpha + \beta) \\ 0 \end{bmatrix} \tag{14}$$

The surface of revolution, which is generated by rotating the circular arc B about the y_e-axis, can be represented in coordinate system S_a as follows (see fig. 7(b)):

$$[r_a] = [M_{ae}] [r_e]$$
$$[n_a] = [L_{ae}] [n_e] \tag{15}$$

where

$$[M_{ae}] = \begin{bmatrix} \cos\Theta_p & 0 & \sin\Theta_p & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\Theta_p & 0 & \cos\Theta_p & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[L_{ae}] = \begin{bmatrix} \cos\Theta_p & 0 & \sin\Theta_p \\ 0 & 1 & 0 \\ -\sin\Theta_p & 0 & \cos\Theta_p \end{bmatrix}$$

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Equations (13) to (15) yield

$$[r_a] = \begin{bmatrix} x_a \\ y_a \\ z_a \\ 1 \end{bmatrix} = \begin{bmatrix} [a_1 \sin \alpha - 2\rho \sin (\beta/2) \sin (\alpha + \beta/2)] \cos \theta_p \\ a_2 \cos \alpha + 2\rho \sin (\beta/2) \cos (\alpha + \beta/2) \\ [a_1 \sin \alpha - 2\rho \sin (\beta/2) \sin (\alpha + \beta/2)] \sin \theta_p \\ 1 \end{bmatrix} \quad (16)$$

$$[n_a] = \begin{bmatrix} \cos (\alpha + \beta) \cos \theta_p \\ \sin (\alpha + \beta) \\ \cos (\alpha + \beta) \sin \theta_p \end{bmatrix}$$

Installing the tool revolute surface in coordinate system S_c is illustrated in figure 8. The coordinate transformation from S_a to S_c is given by equation (9) as

$$[r_c] = [M_{ca}] [r_a] \quad (17)$$

where

$$[M_{ca}] = \begin{bmatrix} \cos (\alpha - \psi_c) & \sin (\alpha - \psi_c) & 0 & -a_1 \sin \psi_c - d \sin (\alpha - \psi_c) \\ -\sin (\alpha - \psi_c) & \cos (\alpha - \psi_c) & 0 & a_1 \cos \psi_c - d \cos (\alpha - \psi_c) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also, we have

$$[n_c] = [L_{ca}] [n_a] \quad (18)$$

where

$$[L_{ca}] = \begin{bmatrix} \cos (\alpha - \psi_c) & \sin (\alpha - \psi_c) & 0 \\ -\sin (\alpha - \psi_c) & \cos (\alpha - \psi_c) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Considering equations (11), (17), and (18) and the pinion generating process shown in figure 2, we can get the following equation of meshing:

$$f(\beta, \theta_p, \phi_p) = \rho \sin \beta \cos \theta_p - a_1 \sin \alpha \sin (\alpha + \beta)(\cos \theta_p - 1)$$

$$+ r_1 \phi_p [\sin (\alpha + \beta) \cos (\alpha - \psi_c) - \cos \theta_p \cos (\alpha + \beta) \sin (\alpha - \psi_c)] = 0 \quad (19)$$

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The pinion tooth surface can be represented in coordinate system S_1 as

$$[r_1] = [M_{1c}] [r_c] = [M_{1f}] [M_{fc}] [r_c] \qquad (20)$$

where the transformation matrices $[M_{fc}]$ and $[M_{1f}]$ are given by

$$[M_{fc}] = \begin{bmatrix} 1 & 0 & 0 & -r_1\phi_p \\ 0 & 1 & 0 & r_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (21)$$

$$[M_{1f}] = \begin{bmatrix} \cos\phi_p & \sin\phi_p & 0 & 0 \\ -\sin\phi_p & \cos\phi_p & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (22)$$

REFERENCES

1. Litvin, F.L.: Theory of Gearing. 2nd ed., Moscow, Nauka, 1968 (in Russia).

2. Litvin, F.L.; and Zhang, J.: Spur Gears: Optimal Geometry, Methods for Generation and Tooth Contact Analysis (TCA) Program. NASA CR-4135, 1988.
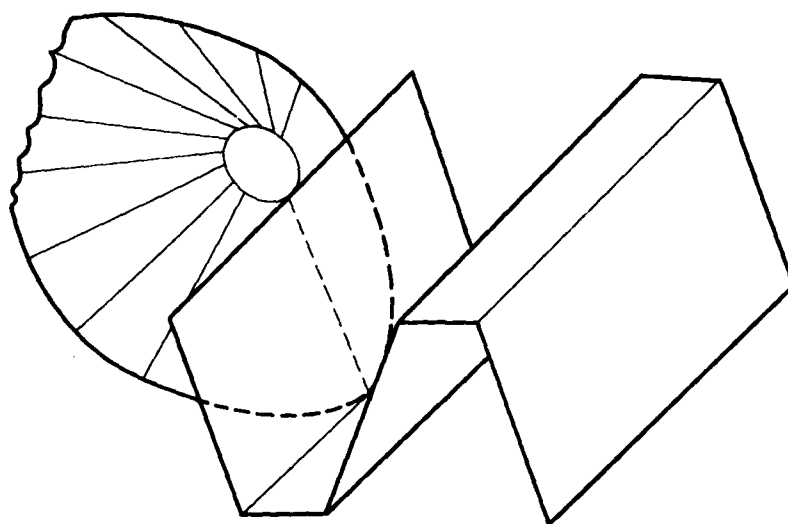
FIGURE 1. - GENERATING SURFACES FOR PINION (CONE) AND GEAR (RACK).
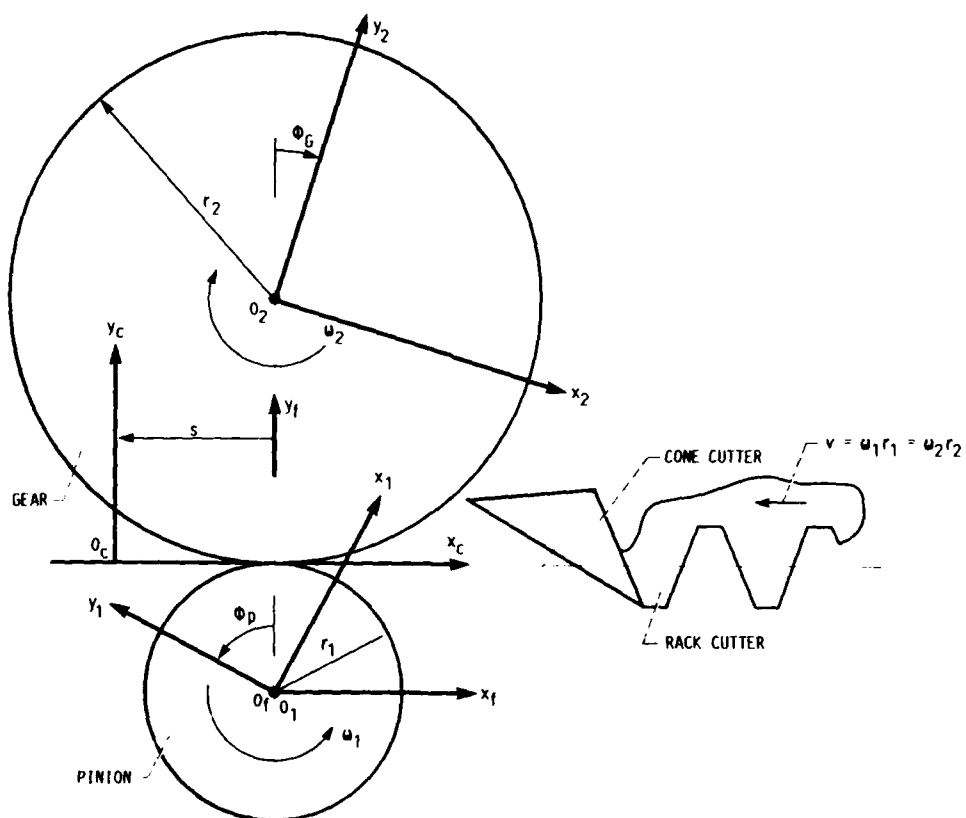
FIGURE 2. - GEOMETRY RELATIONSHIP BETWEEN GENERATING (CUTTERS) AND GENERATED (PINION AND GEAR) SURFACES.

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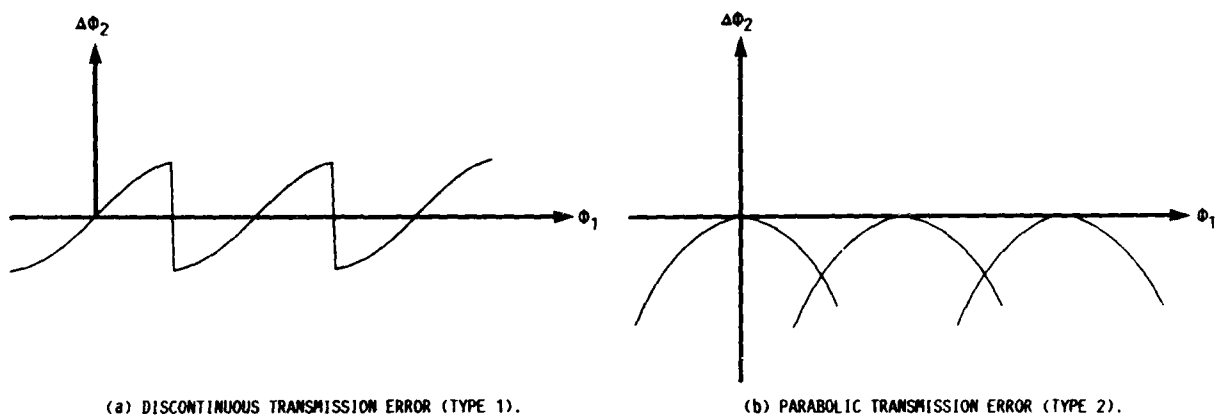
(a) DISCONTINUOUS TRANSMISSION ERROR (TYPE 1).　　　　　　(b) PARABOLIC TRANSMISSION ERROR (TYPE 2).

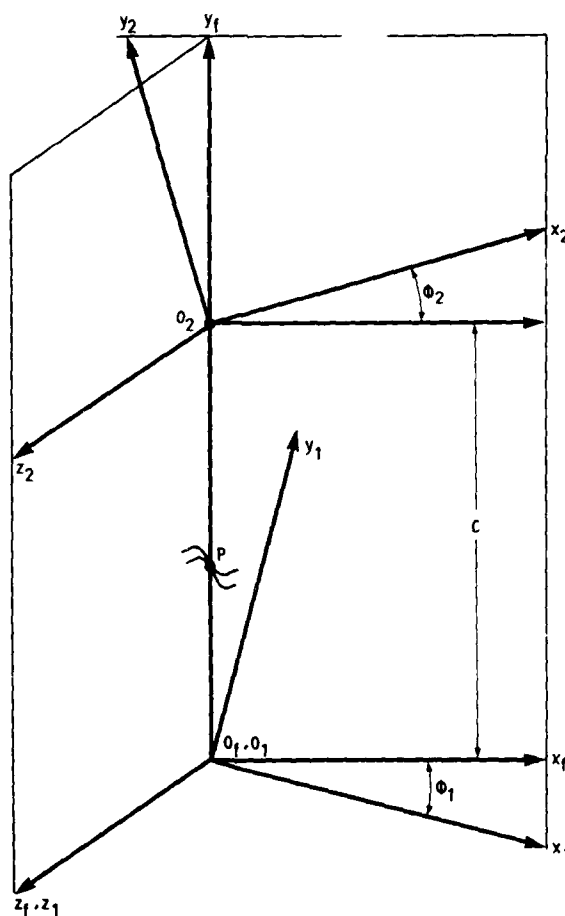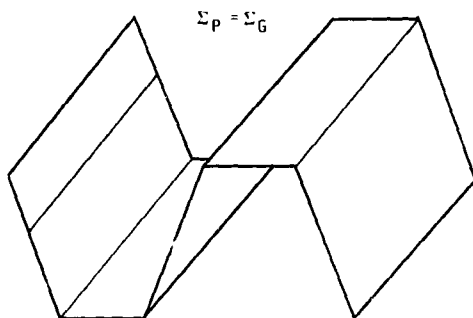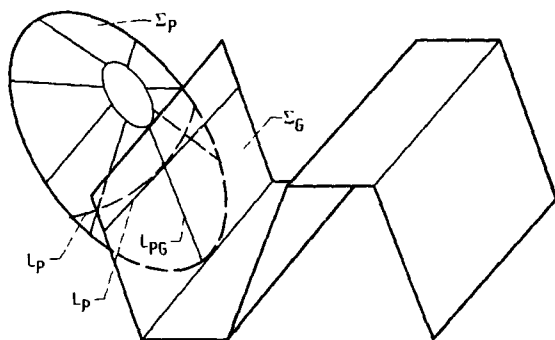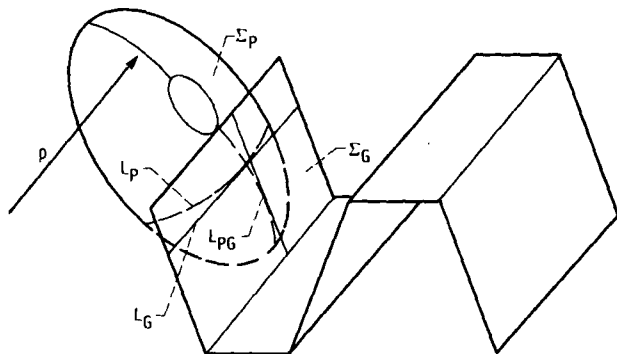FIGURE 3. - TRANSMISSION ERRORTS ($\Delta\Phi_2$) AS FUNCTION OF PINION ROTATION (Φ_1).

FIGURE 4. - COORDINATE SYSTEMS DESCRIBING MESHING OF PINION
AND GEAR.

(a) GENERATING SURFACE FOR INVOLUTE PINION AND GEAR (RACK CUTTERS).

(b) GENERATING SURFACE FOR PINION IS A CONE; GENERATING SURFACE FOR
GEAR IS A RACK.

(c) GENERATING SURFACE FOR PINION IS A SURFACE OF REVOLUTION;
GENERATING SURFACE FOR GEAR IS A RACK.

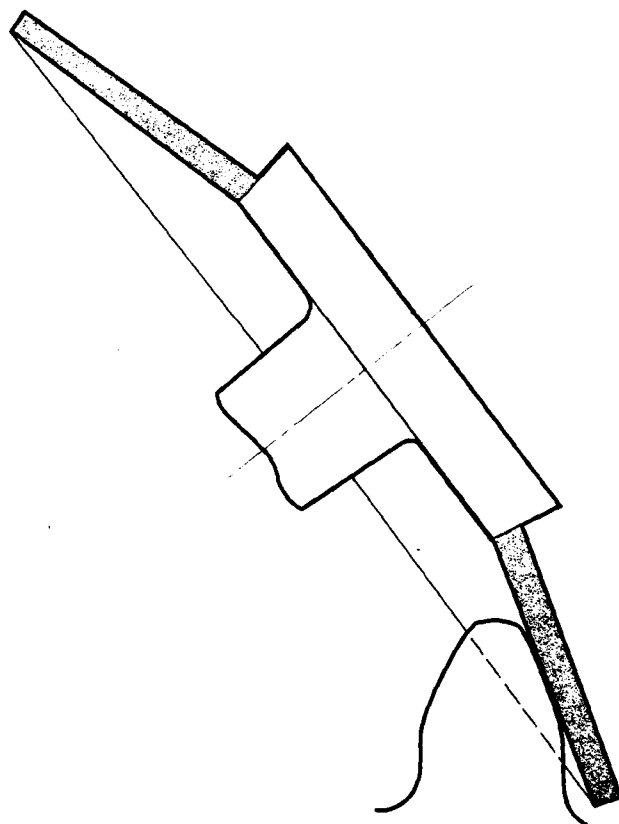FIGURE 5. - THREE DIFFERENT SETS OF PINION AND GEAR GENERATING
SURFACES.

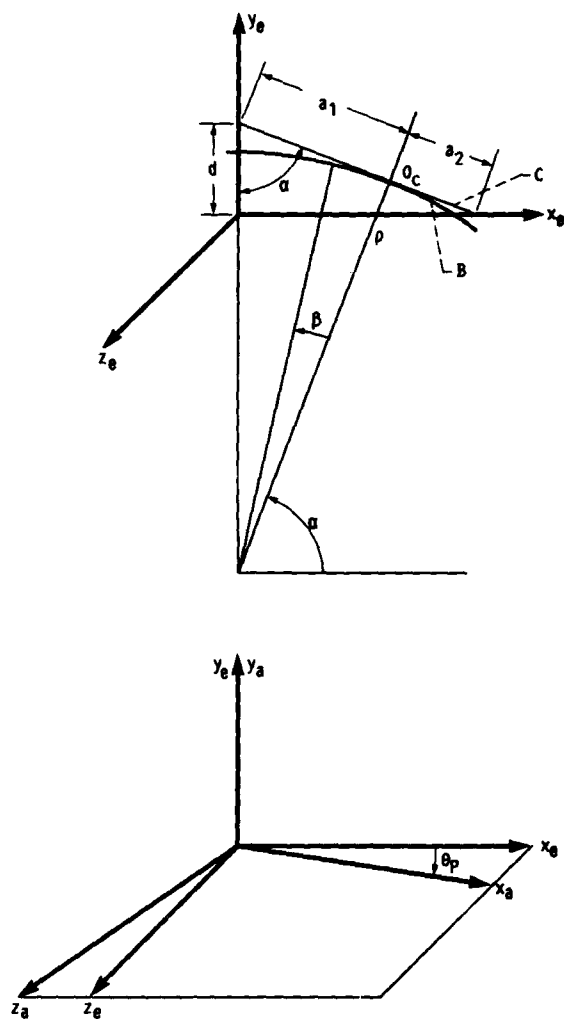FIGURE 6. - PINION GENERATION TOOL AND PINION DURING SURFACE
CROWNING.

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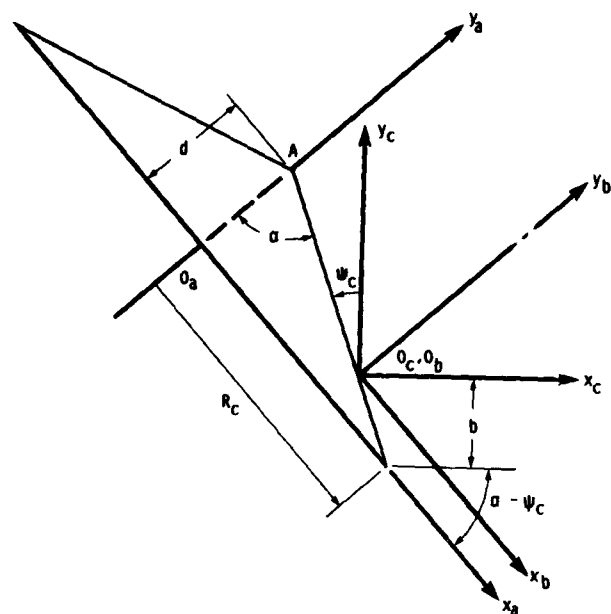FIGURE 7. - COORDINATE SYSTEM DESCRIBING SURFACE OF REVOLUTION
AND RACK CUTTER.

FIGURE 8.- COORDINATE SYSTEM DESCRIBING TRANSFORMATION FROM SYSTEM
S_a TO S_c.

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15. Supplementary Notes

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